Topic: Horizontal asymptotes

Question: Find the horizontal asymptotes of the function.

$$f(x) = \frac{5x - 4}{\sqrt{3x^2 + 2}}$$

Answer choices:

A $y = \frac{5}{3}$

B $y = \frac{5\sqrt{3}}{3}$

C $y = -\frac{5\sqrt{3}}{3}$

D $y = \pm\frac{5\sqrt{3}}{3}$

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Solution: D

Both $y = -5/3$ and $y = 5/3$ are horizontal asymptotes. The limit as x approaches $-\infty$ is

$$\lim_{x \to -\infty} \frac{5x - 4}{\sqrt{3x^2 + 2}}$$

$$\lim_{x \to -\infty} \frac{\frac{5x - 4}{x}}{\frac{\sqrt{3x^2 + 2}}{-\sqrt{x^2}}}$$

$$\lim_{x \to -\infty} \frac{5 - \frac{4}{x}}{-\sqrt{3 + \frac{2}{x^2}}}$$

$$\frac{5 - 0}{-\sqrt{3 + 0}}$$

$$-\frac{5}{\sqrt{3}}$$

The limit as x approaches $+\infty$ is

$$\lim_{x \to \infty} \frac{5x - 4}{\sqrt{3x^2 + 2}}$$

$$\lim_{x \to \infty} \frac{\frac{5x - 4}{x}}{\frac{\sqrt{3x^2 + 2}}{\sqrt{x^2}}}$$

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$$\lim_{x\to\infty}\frac{5-\frac{4}{x}}{\sqrt{3+\frac{2}{x^2}}}$$

$$\frac{5-0}{\sqrt{3+0}}$$

$$\frac{5}{\sqrt{3}}$$

So the horizontal asymptotes are

$$y=\pm\frac{5}{\sqrt{3}}$$

Rationalizing the denominator gives

$$y=\pm\frac{5}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$y=\pm\frac{5\sqrt{3}}{3}$$

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Topic: Horizontal asymptotes

Question: Find the horizontal asymptotes of the function.

$$f(x) = \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x}$$

Answer choices:

A $y = 0$

B $y = -3$

C $y = 2$

D $y = \pm 2$

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Solution: C

The limit as x approaches $+\infty$ of the function is

$$\lim_{x\to\infty} \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x}$$

$$\lim_{x\to\infty} \frac{\frac{4x^3 - 2x^2 + 1}{x^3}}{\frac{2x^3 - 3x}{x^3}}$$

$$\lim_{x\to\infty} \frac{4 - \frac{2}{x} + \frac{1}{x^3}}{2 - \frac{3}{x^2}}$$

$$\frac{4 - 0 + 0}{2 - 0}$$

$$2$$

Therefore, $y = 2$ is the only horizontal asymptote of this function.

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Topic: Horizontal asymptotes

Question: Does the function have a horizontal asymptote? If it does, where is it located?

$$y = \frac{x^5 - x + 6}{x^7 - x^4 + 3x^2 - 1}$$

Answer choices:

A No

B Yes at $y = 5/7$.

C Yes at $y = 0$.

D Yes at $y = 1$.

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Solution: C

To solve, look at the degree of the variable with the largest power in the numerator and compare it to the degree of the variable with the largest power in the denominator.

Given

$$y = \frac{x^5 - x + 6}{x^7 - x^4 + 3x^2 - 1}$$

x^5 is the highest degree term in the numerator, and x^7 is the highest degree term in the denominator.

Since the variable with the largest power in the numerator has a power of 5 and the power of the variable with the largest power in the denominator is 7, there's a horizontal asymptote and it's located at $y = 0$.

Remember, if the degree of the denominator is larger than the degree of the numerator, then the function has a horizontal asymptote at $y = 0$.

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